Topology-II
M. Math - I
End-Semestral Exam
2011-2012

Time: 3 hrs
Max score: 100

Answer all questions.

(1) (a) Show that there does not exist a local homeomorphism from $\mathbb{R}P^2$ to S^2.
(b) Let X be the union of the unit sphere in $\mathbb{R}^3$ with the unit disc in the xy-plane. Find $\pi_1(X)$.

(10+8)

(2) (a) Define the degree of a map $f : S^n \longrightarrow S^n$, for $n \geq 0$.
(b) Show that an even dimensional sphere cannot have a nowhere zero tangent vector field.
(c) Show that any map $f : \mathbb{R}P^{2n} \longrightarrow \mathbb{R}P^{2n}$ has a fixed point.

(2+9+9)

(3) (a) Show that for the subspace $\mathbb{Q} \subset \mathbb{R}$, the relative homology group $H_1(\mathbb{R}, \mathbb{Q})$ is free abelian.
(b) What is the Mayer-Vietoris sequence for singular homology?
(c) Let X be a topological space and let ΣX denote the *suspension* of X, that is the quotient space of $X \times [-1, 1]$ obtained by identifying the subset $X \times 1$ to a point and the subset $X \times (-1)$ to a point. Show that there is an isomorphism

$$\widetilde{H}_p(\Sigma X) \longrightarrow \widetilde{H}_{p-1}(X)$$

for all $p \geq 0$.

(6+4+10)

(4) (a) Prove that $\mathbb{C}P^n$ can be given a CW-structure with one cell in each even dimension less than or equal to $2n$.
(b) Show that the spaces $X = S^1 \times S^1$ and $Y = S^1 \vee S^1 \vee S^2$ have isomorphic homology groups in all dimensions.
(c) Show that the universal covering spaces of X and Y, as in (b), do **not** have isomorphic homology groups in all dimensions.

(6+8+8)

(5) (a) State the universal coefficient theorem for homology.
(b) Compute $H_p(S^2 \vee S^4; \mathbb{Z}_2)$, for $p \geq 0$.

(6+14)